SOLVING SYSTEMS USING TABLES

LESSON 4.3

Determine the solution to a system of equations using tables.

Input-output tables are a tool used in mathematics to display information. Systems of equations can be solved using tables. An input-output table must be created for each equation in the system. The tables can be compared to find an (x, y) pair that is the same in each table. This point represents the solution to the system of equations.

EXPLORE! LARRY'S LANDSCAPING

Larry's Landscaping offers two payment options for his employees. Option #1 offers \$925 per month in salary plus \$25 for every job completed. Option #2 offers \$1,000 every month plus \$10 for every job completed.

Step 1: Write an equation to represent the monthly salary, y, that could be earned for x jobs completed if an employee chooses Option #1.

Step 2: Write an equation to represent the monthly salary, y, that could be earned for x jobs completed if an employee chooses Option #2.

Step 3: Copy the two tables shown below on your own paper. Calculate the monthly salary for an employee under each plan for 0 through 10 jobs.

Option #1

Jobs Completed, x	*Monthly Salary, y*
0	
1	
2	

Option #2

Jobs Completed, x	*Monthly Salary, y*
0	
1	
2	

Step 4: The solution to this system of equations occurs when an employee earns the same amount of money for the same number of jobs. Use your table to determine when this happens. Write your answer in a complete sentence.

Step 5: Verify your answer by substituting the x- and y-values of your solution into the original equations in the system to see if the ordered pair makes each equation true.

Step 6: If an employee thinks he can complete 50 jobs in one month, which payment option should he choose? Explain your answer.

Solving Systems of Linear Equations Using Tables

1. Convert both linear equations in the system to slope-intercept form.
2. Create an input-output table for each equation. Use the same input values for each table.
3. Locate the point in each table where the same pair of input and output values occurs. This is the solution to the system of equations.
4. Verify that the ordered pair is the solution by substituting the x-and y-values into both equations in the system.

EXAMPLE 1

Solve the system of equations using input-output tables. Check the solution.

$$y = -3x + 13 \qquad\qquad 2x + y = 9$$

Solution

Convert both equations to slope-intercept form:

$$y = -3x + 13 \;\downarrow\; \boxed{y = -3x + 13}$$

$$\begin{array}{r} \cancel{2x} + y = 9 \\ \underline{-\cancel{2x} \qquad -2x} \end{array} \;\downarrow\; \boxed{y = 9 - 2x}$$

Create an input-output table for each equation using the same input values.

$y = -3x + 13$

x	y
0	13
1	10
2	7
3	4
4	1
5	−2

$y = 9 - 2x$

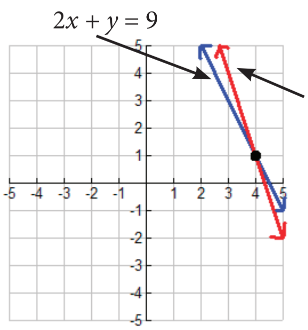
x	y
0	9
1	7
2	5
3	3
4	1
5	−1

The solution is the ordered pair that occurs in both tables.

The solution to the system of equations is (4, 1).

☑ Verify the answer by substituting 4 for x and 1 for y in the original equations.

$$\begin{aligned} y &= -3x + 13 \\ 1 &\stackrel{?}{=} -3(4) + 13 \\ 1 &\stackrel{?}{=} -12 + 13 \\ 1 &= 1 \end{aligned} \qquad\qquad \begin{aligned} 2x + y &= 9 \\ 2(4) + 1 &\stackrel{?}{=} 9 \\ 8 + 1 &\stackrel{?}{=} 9 \\ 9 &= 9 \end{aligned}$$



A solution to a system of equations that is solved using input-output tables can also be verified using a graph. In **Example 1**, the solution to the system of equations is (4, 1). This means the lines intersect at (4, 1).

EXAMPLE 2 **Solve the system of equations using input-output tables. Check the solution.**

$$y = \frac{1}{2}x + 4 \qquad\qquad y = -x + 1$$

SOLUTION

Both equations are in slope-intercept form. Create an input-output table for each equation using the same input values.

$y = \frac{1}{2}x + 4$

x	y
0	4
1	$4\frac{1}{2}$
2	5
3	$5\frac{1}{2}$

y-values are getting larger.

$y = -x + 1$

x	y
0	1
1	0
2	−1
3	−2

y-values are getting smaller.

The y-values are going in opposite directions. The solution must have a negative x-value. Use input values that are negative to find the solution.

$y = \frac{1}{2}x + 4$

x	y
−1	$3\frac{1}{2}$
−2	3
−3	$2\frac{1}{2}$
−4	2

$y = -x + 1$

x	y
−1	2
−2	3
−3	4
−4	5

The solution to the system of equations is (−2, 3).

☑ Verify the answer by substituting −2 for x and 3 for y in the original equations.

$$y = \frac{1}{2}x + 4 \qquad\qquad y = -x + 1$$
$$3 \stackrel{?}{=} \frac{1}{2}(-2) + 4 \qquad\qquad 3 \stackrel{?}{=} -(-2) + 1$$
$$3 \stackrel{?}{=} -1 + 4 \qquad\qquad 3 \stackrel{?}{=} 2 + 1$$
$$3 = 3 \qquad\qquad 3 = 3$$

EXERCISES

Solve each system of equations using the given input-output tables. Show all work necessary to justify your answer.

1. $y = 3x - 1$

x	y
0	
1	
2	
3	

$y = -2x + 4$

x	y
0	
1	
2	
3	

2. $y = x + 6$

x	y
−3	
−2	
−1	
0	

$y = \frac{1}{2}x + 5$

x	y
−3	
−2	
−1	
0	

3. Abe created input-output tables with input-values of 0, 1, 2, 3 and 4 to solve his system of equations. After looking at his output values, he realized he needed to try negative input values. What do you think he noticed about his output values?

Solve each system of equations using input-output tables. Show all work necessary to justify your answer.

4. $y = 5x - 6$
$y = -2x + 15$

5. $y = x + 2$
$y = 2x + 1$

6. $y = 3x + 4$
$y = 2x + 14$

7. $y = \frac{1}{2}x$
$y = 3x + 10$

8. $2x + y = 3$
$y - 3x = 23$

9. $-4x + 2y = 12$
$y = -5x + 6$

10. Two submarines were headed toward one another. One followed the path represented by the equation $y = 4x + 7$. The other submarine followed the path represented by the equation $y = 3x + 12$. Let x represent the number of minutes the submarines have been in motion and y represent the distance each is from the submarine base.

a. Solve the system of equations using two input-output tables to determine when the submarines' paths will cross.

b. Explain how you know your answer is correct.

11. Carlos put \$100 in a savings account at the beginning of the year. At the end of each month, he added \$15 to the account. Ana put \$400 in her savings account at the beginning of the year. At the end of each month, she took \$35 out of her account. Let x represent the number of months which have passed and y represent the amount in each savings account.

a. Write an equation to represent the amount in Carlos' savings account.

b. Write an equation to represent the amount in Ana's savings account.

c. Copy and complete the input-output tables through 8 months.

Carlos' Savings Account Balance

Months, x	*Total Savings, y*
0	
1	
2	

Ana's Savings Account Balance

Months, x	*Total Savings, y*
0	
1	
2	

d. When will Carlos and Ana have the same amount in their savings accounts? How much will they each have at this time?

12. Solve each system of equations using input-output tables. Verify each solution by graphing the two equations.

a. $y = \frac{1}{2}x - 1$
$y = -x + 5$

b. $y = -2x - 3$
$y = x - 9$

13. Joshua's profits, P, for his lawn-mowing business are represented by the equation $P = 16m - 52$ where m is the number of lawns he has mowed. Serj also runs a lawn-mowing business. His profits can be calculated using the equation $P = 14m - 40$. How many lawns do they have to mow to make the same amount of profit? Explain how you know your answer is correct.

REVIEW

Solve each equation. Show all work necessary to justify your answer.

14. $8x - 10 = -2x + 60$

15. $\frac{2}{3}x + 7 = \frac{4}{3}x + 6$

16. $4x + 2 = 5x + 7$

17. $-2x = 6x + 40$

18. $x + 3 = \frac{1}{2}x + 1$

19. $3.2x - 12 = 4.7x - 9$

Tic-Tac-Toe ~ Different Systems

Systems of equations can include equations that are not linear. In this activity, you will be finding the solutions to systems of equations containing a linear equation and a quadratic equation. Each system will have 2 solutions. You may use graphing or input-output tables to find the solutions.

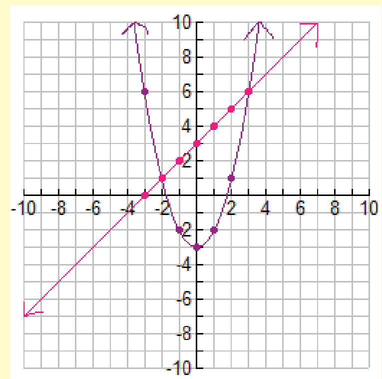
For example: $y = x + 3$
$y = x^2 - 3$

$y = x + 3$

x	y
−3	0
−2	1
−1	2
0	3
1	4
2	5
3	6

$y = x^2 - 3$

x	y
−3	6
−2	1
−1	−2
0	−3
1	−2
2	1
3	6



SOLUTIONS: (−2, 1) and (3, 6)

Find the two solutions to each system of equations. Show all work.

1. $y = x^2$
$y = 2x + 3$

2. $y = -x^2 + 1$
$y = x - 5$

3. $y = 3x + 1$
$y = x^2 + 1$

Tic-Tac-Toe ~ Math Dictionary

Create a "Linear Equations" Dictionary. Locate all of the vocabulary words from the first four Blocks in this textbook. Alphabetize the list of words and design a dictionary. The dictionary should include each word, spelled correctly, along with the definition. If appropriate, a diagram or illustration can be included.